

Holographic entanglement entropy in open-closed string duality

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We study minimal co-dimension-2 surfaces in the asymptotically flat background of extremal 3-brane solutions in ten-dimensional type IIB supergravity. A conjectured open-closed string duality combined with the Ryu-Takayanagi prescription implies that the area of the surfaces we consider could be interpreted as the entanglement entropy of a dual (3+1)-dimensional large- N , strongly-coupled open string field theory on D3-branes. As the size of the surface is varied we observe a transition from a volume law to an area law in agreement with expectations from non-locality in an open string field theory. Some of the specifics of this transition bear a qualitative resemblance with the behaviour of holographic entanglement entropy in non-commutative super-Yang-Mills theory.

INTRODUCTION

A holographic correspondence between theories of gravity in asymptotically flat backgrounds and open string theories, generalizing the AdS/CFT correspondence beyond the near-horizon limit [1], has been conjectured by several authors in the past, see e.g. [2–16]. For D3-branes in ten-dimensional type IIB supergravity the conjecture proposes a holographic relation between closed string theory in 3-brane solutions and the large- N , strongly-coupled $U(N)$ string field theory on D3-branes.

More recently in [15] the following points were put forward/emphasized: (i) General open-closed string dualities may arise for string field theories on D-brane setups (with suitable co-dimension) in generic closed string backgrounds. The asymptotic closed string background plays the role of an arbitrary external source for open string fields. (ii) The absence of a near-horizon decoupling limit in these dualities may be justifiable as a consequence of the open string completeness conjecture by A. Sen [17, 18]. Non-trivial checks of this conjecture were presented in open string tachyon condensation and non-critical string theory. (iii) In this more general version of the holographic principle it is proposed that holography works as a *tomographic* principle, where different open string field theories capture/reconstruct holographically different subsectors of closed string theory/gravity.

Finding evidence in favor of this proposal, and setting up a solid holographic dictionary, is a highly non-trivial task, largely hindered at the present stage by the lack of powerful computational tools in interacting open string field theories. This is unlike the AdS/CFT correspondence where many non-trivial checks can be found by matching explicit (super)gravity computations to corresponding computations in strongly-coupled large- N quantum field theories. With such technical limitations it is useful to identify structures in classical gravity that reproduce familiar features from open string theory. In [15] (see also [19]) we focused on the long-wavelength dynamics of asymptotically flat p -brane solutions, and argued that the sector related to the abelian (center-of-

mass motion, singleton) dynamics of the branes could be identified in gravity within the blackfold approach [20, 21], and systematically recast in terms of the abelian Dirac-Born-Infeld (DBI) effective action. The latter is a characteristic feature of open string theory. For other recent discussions of the emergence of the DBI action from (super)gravity we refer the reader to [16, 22, 23].

In this note we are looking for a quantity that has the power to probe more efficiently the full non-abelian sector of the dual $U(N)$ string field theory. For concreteness, we will focus on the canonical example of D3-branes in flat space. The quantity we would like to consider is entanglement entropy (EE). This is an observable that can probe interactions and degrees of freedom across different scales. It has been proposed to have a simple holographic manifestation in gravity as the area of a corresponding minimal co-dimension-2 surface [24, 25]. Although this proposal is better understood in the case of AdS space [26–29], here we will assume that the prescription is more generally valid and applies also to asymptotically flat spacetimes. Our main goal is to examine the properties of holographic entanglement entropy (HEE) in asymptotically flat 3-brane solutions where open-closed string duality could operate according to the above-mentioned conjectures.

A clear feature of open string theory we should be looking for is non-locality. This is manifest in EE as a volume law, rather than an area law, which is characteristic of local quantum field theory [30–32]. The volume law in HEE has been noted before in a related context in a proposal for flat space holography in [33]. However, unlike [33] in this note we are not considering HEE solely in flat space; we are considering it in a solution that interpolates between a near-horizon AdS space and the asymptotic flat space, where we have a more specific conjecture for the anticipated dual non-gravitational theory. Besides the volume law, which is mainly due to flat space (in analogy to [33]), our case also exhibits a transition to the more standard area law as the size of the entangling region is varied. We argue that the main features of this result are consistent with expectations from an interacting open string field theory.

The volume law has also been noted in the (3+1)-dimensional holographic duals of the dipole theory and non-commutative super-Yang-Mills (NCSYM) theory [30, 34, 35]. We will observe that the asymptotically flat 3-brane HEE exhibits similarities with the HEE of the NCSYM theory.

MAIN RESULT

We consider the HEE, \mathcal{S}_{AS} , of a prospective dual (3 + 1)-dimensional open string field theory on a straight belt

$$A_S = \left\{ x^i \middle| x^1 \in \left[-\frac{\ell}{2}, \frac{\ell}{2} \right], x^2, x^3 \in \left[-\frac{L}{2}, \frac{L}{2} \right] \right\}. \quad (1)$$

L is an infrared cutoff that is taken eventually to infinity. ℓ is the tunable width of the belt. We will also introduce an ultraviolet length cutoff ε that is kept fixed throughout the computation. We are mainly interested in the regime

$$\varepsilon \ll (4\pi\lambda)^{\frac{1}{4}} \ell_s, \quad (2)$$

where the asymptotic radial cutoff r_ε in the gravitational bulk is outside the near-horizon region ($r_H \ll r_\varepsilon = \frac{r_H^2}{\varepsilon}$ —see eq. (6) for the definition of r_H). $\ell_s = \sqrt{\alpha'}$ is the string length. When the cutoff is well inside the near-horizon throat we recover the familiar divergent piece of the $\mathcal{N} = 4$ SYM EE, $\mathcal{S}_{AS} \simeq \frac{N^2}{2\pi} \frac{L^2}{\varepsilon^2}$, which is less interesting for our purposes.

The main result of this note is the following prediction for the HEE of the large- N string field theory on D3-branes in the regime (2)

$$\frac{\mathcal{S}_{AS}(\ell)}{L^2} = \begin{cases} \frac{N^2 \sqrt{\lambda}}{\sqrt{\pi}} \frac{\alpha' \ell}{\varepsilon^5} + \dots, & \ell \ll \ell_c \\ \frac{2N^2 \lambda}{3} \frac{\alpha'^2}{\varepsilon^6} + \dots, & \ell \gg \ell_c \end{cases}. \quad (3)$$

$\ell_c \sim \sqrt{\lambda} \frac{\alpha'}{\varepsilon}$ is a critical length where a transition between different branches of extremal co-dimension-2 surfaces occurs. It is estimated numerically in eq. (13) and with an analytic extrapolation in eq. (18). The dots indicate less divergent terms.

Eq. (3) exhibits a transition between a short distance regime with a characteristic volume dependence to a long distance regime with the more standard area dependence. We find evidence that this is a robust feature of the system independent of the choice of the geometry of the entangling region A_S . A similar computation of the HEE for the cylinder

$$A_S = \left\{ x^i \middle| (x^1)^2 + (x^2)^2 = \ell^2, x^3 \in \left[-\frac{L}{2}, \frac{L}{2} \right] \right\} \quad (4)$$

verifies the same type of transition (see eq. (22)) consistent with an interpretation based on open-closed string duality. The physical content of eq. (3) will be discussed

further in the last section. In the next section we explain how (3) is derived from gravity. For economy and clarity of the presentation we will focus exclusively on the derivation of the HEE on the straight belt geometry (1).

HEE FROM CLASSICAL GRAVITY

We focus on N parallel overlapping D3-branes in flat space in ten-dimensional type IIB string theory. In the large- N 't Hooft limit, where $\lambda = g_s N$ is kept fixed and large (g_s being the string coupling), this setup is most conveniently described by the extremal 3-brane supergravity solution

$$ds^2 = H^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/2} (dr^2 + r^2 d\Omega_5^2), \quad (5)$$

$$e^\Phi = g_s, \quad C_4 = g_s^{-1} (H^{-1} - 1) dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3.$$

The dilaton Φ is constant and the 4-form C_4 yields a self-dual 5-form flux. The solution is expressed here in the string frame. The function H is

$$H(r) = 1 + \frac{r_H^4}{r^4}, \quad r_H^4 = 4\pi\lambda\alpha'^2. \quad (6)$$

$r \rightarrow \infty$ is the asymptotic flat space region.

In what follows we will express all length scales of the problem in terms of the length scale r_H that is intrinsic to the solution (5). In particular, for the quantities in (1) we set $r = u r_H$, and $\ell = l r_H$, $L = \mathfrak{L} r_H$, $\varepsilon = \epsilon r_H$, where u, l, \mathfrak{L} and ϵ are now dimensionless quantities.

Following the Ryu-Takayanagi prescription [24, 36, 37] we are looking for a constant time, co-dimension-2 minimal surface \bar{A}_S in the background (5) that asymptotes (at large radial distance u) to the boundary of the region (1). The volume of this minimal surface,

$$\text{Vol}(\bar{A}_S) = \int d^8 \sigma e^{-2\Phi} \sqrt{G_{ind}^{(8)}}, \quad (7)$$

can be used to express the HEE as

$$\mathcal{S}_{AS} = \frac{\text{Vol}(\bar{A}_S)}{32\pi^6 (\alpha')^4}. \quad (8)$$

In eq. (7) $G_{ind}^{(8)}$ is the determinant of the induced metric on the co-dimension-2 surface \bar{A}_S . Wrapping this surface around the 5-sphere of the background (5) and the directions x^2, x^3 , and assuming a single dependence on the coordinate $x \equiv \frac{x^1}{r_H}$, we can express it as a solution $u(x)$ that extremizes the volume (7), written more explicitly as

$$\text{Vol}(\bar{A}_S) = \frac{\pi^3 r_H^8 \mathfrak{L}^2}{g_s^2} \int dx u^5 H^{1/2} \sqrt{1 + H(\dot{u})^2}, \quad (9)$$

where $H = 1 + u^{-4}$ and $\dot{u} = \frac{\partial u}{\partial x}$. The extremization equations of (9) can be recast as the conservation equation

$$u^5 \sqrt{\frac{1 + u^{-4}}{1 + (1 + u^{-4}) \dot{u}^2}} = u_*^5 \sqrt{1 + u_*^{-4}} \equiv c, \quad (10)$$

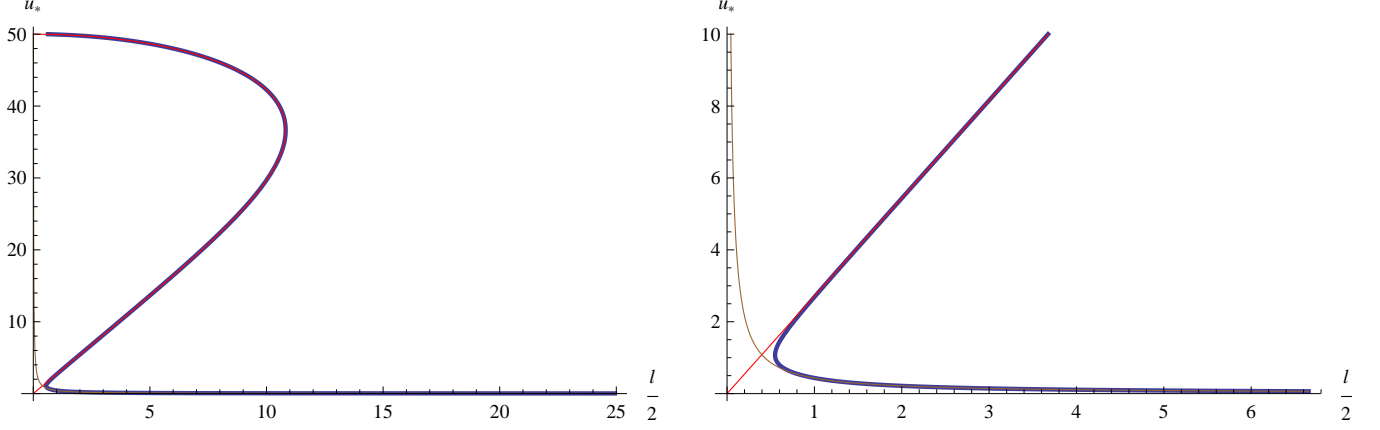


FIG. 1. A plot of the turning point u_* of the solutions of eq. (10) as a function of the dimensionless belt half-width $\frac{l}{2}$. Both plots represent the solution for the same cutoff value $\epsilon = 0.02$ (blue thick line). The right plot zooms around the point where the middle and bottom branches meet. The red curve is an analytic prediction based on the approximation (14). The brown curve is an analytic prediction based on the approximation (19).

where u_* is the turning point of the surface in the bulk. c is essentially the value of the conserved Hamiltonian of the system (9). We solve the non-linear first-order differential equation (10) with an explicit UV cutoff ϵ

$$u\left(x = \pm \frac{l}{2}\right) = \frac{1}{\epsilon}. \quad (11)$$

As was noted in previous work [35], it is important to keep the cutoff fixed throughout the computation, otherwise important branches of the solution can be missed as $\epsilon \rightarrow 0$.

After a few trivial algebraic manipulations, the HEE (8) evaluated on the on-shell profile of the function $u(x)$ can be written as

$$\mathcal{S}_{As} = \frac{\mathfrak{L}^2 N^2}{\pi c} \int_{u_*}^{\frac{1}{\epsilon}} du u^{10} \sqrt{\frac{(1+u^{-4})^3}{u^{10} c^{-2} (1+u^{-4}) - 1}}. \quad (12)$$

The equations (10), (11) are easily solved numerically. The turning point u_* as a function of the belt width l is plotted in Fig. 1 for the cutoff value $\frac{1}{\epsilon} = 50$, or $\epsilon = 0.02$ (other small values of ϵ were checked to exhibit the same behavior). We observe an intermediate range of l , where there are three separate branches of external surface solutions, let us call them upper, middle and bottom branches for decreasing values of u_* . In the upper branch the extremal surface is mainly embedded in the asymptotic flat space region. In the lower branch the extremal surface is well embedded inside the AdS throat. We will analyse each of these branches analytically with perturbative methods in a moment.

In Fig. 2 we plot the value of the HEE for each of these branches. We observe that the upper branch is the dominant (or single) branch for all values of l in the interval $[0, l_c)$. For $l > l_c$ the bottom (AdS) branch takes over and dominates. At very large values of l this branch

is the only existing branch. The existence of a similar pattern of three co-existing extremal surfaces was also observed in the NCSYM theory in [35].

Numerically, we observe that the critical width is

$$l_c \sim \frac{0.4}{\epsilon} \Leftrightarrow \ell_c \sim \frac{0.4 r_H^2}{\varepsilon} = 0.8 \sqrt{\pi \lambda} \frac{\ell_s}{\varepsilon} \ell_s. \quad (13)$$

The explicit presence of the UV cutoff scale ε in ℓ_c was also noted in the case of the NCSYM theory [30, 34, 35]. An analytic rough estimate of ℓ_c will be discussed soon.

We proceed to analyse each of these branches analytically with approximations performed in two opposite regimes: $u_* \sim \frac{1}{\epsilon} \gg 1$ and $u_* \ll \frac{1}{\epsilon}$.

Approximations in the flat space regime: $u_* \gg 1$. In this regime the function $H(u)$ is 1 to first approximation and $c \sim u_*^5$. Then, we can easily find an analytic solution for $u(x)$ in the form

$$x = -\frac{\sqrt{\pi} \Gamma\left(\frac{2}{5}\right) u_*}{\Gamma\left(-\frac{1}{10}\right)} - \frac{u_*^5}{4u^4} {}_2F_1\left(\frac{2}{5}, \frac{1}{2}, \frac{7}{5}, \frac{u_*^{10}}{u^{10}}\right). \quad (14)$$

The relation between l and $\frac{1}{\epsilon}$ follows immediately from this equation by implementing the boundary condition (11). We notice that the resulting curve (red curve in both plots in Fig. 1) agrees very well with the numerical solution in both the upper and middle branches and starts deviating only when $u_* \sim 1$ and $l \sim 1$.

Under the same assumptions, for the HEE we obtain the expression

$$\mathcal{S}_{As} = \frac{\mathfrak{L}^2 N^2}{\pi} \frac{u_*^6}{30} \left[\frac{3\sqrt{\pi} \Gamma\left(-\frac{3}{5}\right)}{\Gamma\left(-\frac{1}{10}\right)} + 5 \left(\frac{1}{\epsilon u_*}\right)^6 {}_2F_1\left(-\frac{3}{5}, \frac{1}{2}, \frac{2}{5}, (\epsilon u_*)^{10}\right) \right]. \quad (15)$$

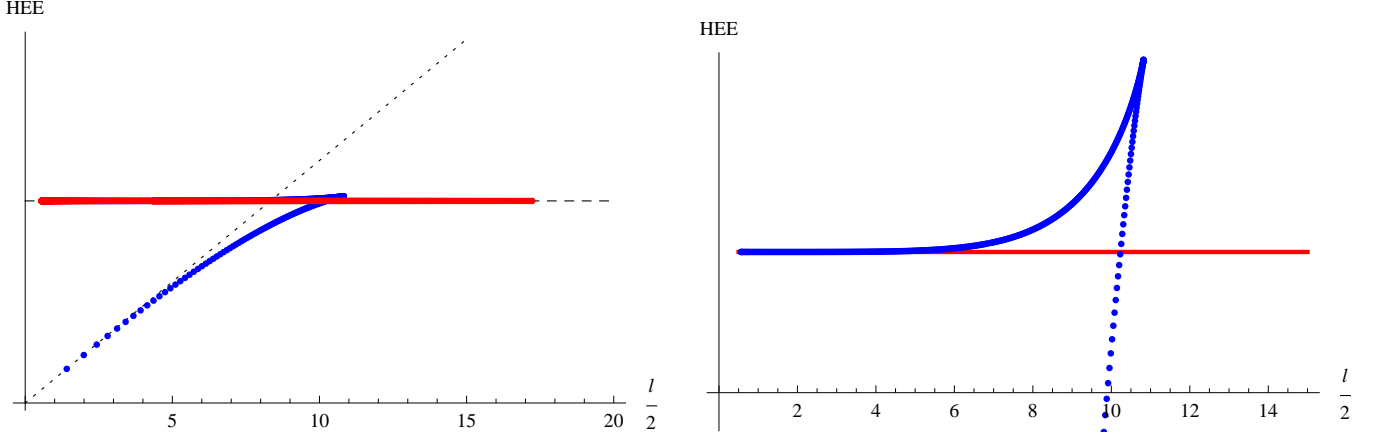


FIG. 2. A plot of the profile of the HEE as a function of the belt half-width $\frac{l}{2}$. The same cutoff value $\epsilon = 0.02$ is used in this figure as in Fig. 1. The right plot zooms into the region of the transition at $l = l_c$ of the left plot. The dotted line that passes through the origin in the left plot depicts the volume law (16), which is linear in l . The dashed horizontal line depicts the area law (17). The blue points refer to the numerical data for the upper and middle branches, and the red points to the data for the bottom branch.

It is not hard to show by a series expansion of this equation that when $u_* \sim \frac{1}{\epsilon}$ to leading order

$$\mathcal{S}_{A_S} \simeq \frac{N^2}{2\pi} \frac{\mathfrak{L}^2 l}{\epsilon^5}, \quad (16)$$

which is proportional to the volume of the belt A_S , and represents the leading contribution to the HEE when $l \ll l_c$. This result provides the upper line term on the r.h.s. of eq. (3)). The linear expression in l , (16), is depicted in the left plot of Fig. 2 by the dotted line that goes through the origin.

In a different limit, where $u_* \ll \frac{1}{\epsilon}$, we can deduce from the series expansion of the expression (15) the leading HEE contribution

$$\mathcal{S}_{A_S} \simeq \frac{N^2}{6\pi} \frac{\mathfrak{L}^2}{\epsilon^6}. \quad (17)$$

Strictly speaking this result is valid only under the additional assumption $u_* \gg 1$ along the subdominant middle branch. It is clear, however, from the numerical data depicted in Figs. 1, 2 that the solution (14) works impressively well even for order 1 values of u_* . It is also clear from the data of Fig. 2 that once the middle branch asymptotes to the bottom branch, it reaches the value (17) which remains constant as l is increased. This constant is the same as the leading divergence of the bottom branch (see red curve in the plots of Fig. 2).

This observation allows us to obtain a rough estimate of l_c by equating the HEEs in eqs. (16) and (17)

$$\frac{N^2}{2\pi} \frac{\mathfrak{L}^2 l_c}{\epsilon^5} \sim \frac{N^2}{6\pi} \frac{\mathfrak{L}^2}{\epsilon^6} \Leftrightarrow l_c \sim \frac{1}{3\epsilon}. \quad (18)$$

Comparing with (13) we observe that the factors ~ 0.4 (from the numerics) and $\frac{1}{3} \sim 0.3$ (from the analytic approximation), compare well with each other.

Information from the AdS regime: $u_* \ll 1$. Assume for a moment that we take the radial cutoff r_ϵ inside the near-horizon region, i.e. we take $r_\epsilon \ll r_H$. In this regime we recover the minimal surface as a deformation of the more familiar $AdS_5 \times S^5$ results. The function $H(u)$ is to first approximation u^{-4} and $c \sim u_*^3$. The analytic solution for the function $u(x)$ in this case implies at the cutoff

$$\frac{\ell}{2} = \frac{1}{u_*} \left[\frac{\sqrt{\pi} \Gamma(\frac{2}{3})}{\Gamma(\frac{1}{6})} - (\epsilon u_*)^6 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, (\epsilon u_*)^6\right) \right]. \quad (19)$$

One can see from the numerical solutions of the minimal surface in the opposite regime $r_H \ll r_\epsilon$ (i.e. the regime (2) of interest in this note) that the position $x(u)$ of the surface moves very slowly as u decreases until the surface reaches the near-horizon throat. As a result, the equation (19) is not only a good approximation in the AdS regime, $r_\epsilon \ll r_H$, but also in the regime (2). This result is clearly visible in the numerical results of Fig. 1, where the brown curve depicting (19) is in excellent agreement with the bottom branch from the region $u \sim 1$, $l \sim 1$ and towards higher values of l .

INTERPRETATION OF RESULTS

The above-mentioned conjectures of open-closed string duality imply, if correct, that the HEE (3) is holographically related to the EE of a dual large- N open string field theory. We cannot verify (3) directly in a strongly interacting open string field theory, but it is still natural to ask if a transition like (3) could be envisaged in an open string theory. Here we would like to argue that the answer to this question is naturally a positive one.

Let us start with the estimate of the transition scale $\ell_c \sim \sqrt{\lambda} \frac{\alpha'}{\varepsilon}$. The $\sqrt{\lambda}$ factor could clearly be an effect of the strong 't Hooft coupling limit. In a weakly coupled string theory one would anticipate a transition scale of the form $\ell_c \sim \frac{\alpha'}{\varepsilon}$. Could this be consistent? From previous discussions of EE in non-local theories (see e.g. [30]) we know that ℓ_c is naturally associated with the scale of non-locality. In the case of an open string theory the characteristic size of a rotating string with energy E is $\ell_E \sim E\alpha'$ [38]. Consequently, in the presence of a UV cutoff ε^{-1} the maximum value of ℓ_E , which should be associated with the non-locality scale, is $\ell_{\varepsilon^{-1}} = \ell_c \sim \frac{\alpha'}{\varepsilon}$ exactly as anticipated above. The relevance of this scale, instead of ℓ_s , is a sign of UV/IR mixing as pointed out in the context of NCSYM theories in [30].

We can now ask about the more precise transition implied by (3). First, we notice that eq. (3) can be recast (up to an overall numerical factor for each line on the r.h.s.) as

$$\mathcal{S}_{A_S}(\ell) \sim \begin{cases} N^2 \frac{L^2 \ell_c \ell}{\varepsilon^4} + \dots & , \ell \ll \ell_c \\ N^2 \frac{L^2 \ell_c^2}{\varepsilon^4} + \dots & , \ell \gg \ell_c \end{cases} . \quad (20)$$

Moreover, in this language the condition (2) becomes simply $\varepsilon \ll \ell_c$. In a weakly coupled open string theory with a UV cutoff ε^{-1} and $U(N)$ Chan-Paton indices one expects the following behaviour of the EE. When $\ell \ll \ell_c$, all the degrees of freedom in the region A_S can interact (and entangle) with the outside degrees of freedom. In a unit cell of volume ε^3 there are roughly $N^2 \frac{\ell_c}{\varepsilon}$ degrees of freedom, so one expects the EE to scale as

$$\mathcal{S}_{A_S} \sim \ell L^2 \cdot N^2 \frac{\ell_c}{\varepsilon} \cdot \frac{1}{\varepsilon^3} = N^2 \frac{\ell_c \ell}{\varepsilon^4} \quad (21)$$

reproducing the volume law in the first line of the r.h.s. of eq. (20). Similarly, when $\ell \gg \ell_c$ only degrees of freedom inside a strip of size ℓ_c around the boundary ∂A_S can at most entangle with the outside, which leads to the area scaling of the second line of the r.h.s. of eq. (20). Exactly the same type of scaling and transition (with α' replaced by the non-commutativity scale θ) was argued in the NCSYM theory [30, 35]. Of course, there are also important differences with the NCSYM theory, which exhibits anisotropy in certain spacetime directions.

The above interpretation suggests that the transition (20), from a volume dependence to an area dependence, is a feature independent of the geometry of the region A_S . In agreement with this expectation, we have verified for the cylinder geometry (4) that the HEE exhibits the leading divergent terms

$$\mathcal{S}_{A_S}(\ell) \sim \begin{cases} N^2 \frac{L \ell_c \ell^2}{\varepsilon^4} + \dots & , \ell \ll \ell_c \\ N^2 \frac{L \ell_c^2 \ell}{\varepsilon^4} + \dots & , \ell \gg \ell_c \end{cases} \quad (22)$$

with a critical length ℓ_c again of the order $\sqrt{\lambda} \frac{\alpha'}{\varepsilon}$.

It would be interesting to reproduce the above expectations with an explicit computation in weakly coupled open string theory, perhaps along similar lines to the computation performed in [39]. A related computation worth exploring has to do with the corrections of the entanglement entropy of $\mathcal{N} = 4$ SYM theory in the presence of irrelevant deformations induced by open string theory. In the bulk this involves a computation in a regime where the radial cutoff r_ε is comparable to r_H . In this paper we focused on the regime $r_\varepsilon \gg r_H$.

Finally, note that in the computations of this paper the volume law came essentially from the effects of the asymptotic flat space —the scaling of eq. (16) was derived exclusively in flat space. We would like to emphasize two points related to this feature that are conceptually close to the discussion of holography as a tomographic principle in [15]. First, we note that although flat space itself does not have a preferred radial direction, the 3-brane solution naturally defines one via the splitting $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + r^2 d\Omega_5^2$ that leads to the minimal co-dimension-2 surface we considered. For a p -brane solution of different worldvolume dimension a different transverse space would be chosen leading to another minimal surface. Second, notice that we would not have obtained sensible results had we restricted only to the flat space part of our computation. In flat space the minimal surface would not exhibit the bottom branch of Fig. 1 and above some width ℓ_{max} no minimal surface would exist. In the 3-brane setup this potential issue is remedied by the non-trivial geometry in the bulk that deviates from flat space.

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